List of Refereed Publications

Jonathan Chappelon

1. A universal sequence of integers generating balanced Steinhaus figures modulo an odd number, accepted for publication in J. Combin. Theory Ser. A, 30 pages.

Abstract : In this paper, we partially solve an open problem, due to J.C. Molluzzo in 1976, on the existence of balanced Steinhaus triangles modulo a positive integer n, that are Steinhaus triangles containing all the elements of $\mathbb{Z}/n\mathbb{Z}$ with the same multiplicity. For every odd number n, we build an orbit in $\mathbb{Z}/n\mathbb{Z}$, by the linear cellular automaton generating the Pascal triangle modulo n, which contains infinitely many balanced Steinhaus triangles. This orbit, in $\mathbb{Z}/n\mathbb{Z}$, is obtained from an integer sequence called the universal sequence. We show that there exist balanced Steinhaus triangles for at least 2/3 of the admissible sizes, in the case where n is an odd prime power. Other balanced Steinhaus figures, such as Steinhaus trapezoids, generalized Pascal triangles, Pascal trapezoids or lozenges, also appear in the orbit of the universal sequence modulo n odd. We prove the existence of balanced generalized Pascal triangles for at least 2/3 of the admissible sizes, in the case where n is an odd prime power, and the existence of balanced lozenges for all admissible sizes, in the case where n is a square-free odd number.

MSC2010: 05B30, 11B50.

Keywords: Molluzzo problem, balanced Steinhaus figure, universal sequence, Steinhaus figure, Steinhaus triangle, Pascal triangle.

2. On the multiplicative order of a^n modulo n, J. Integer Seq. **13** (2010), Article 10.2.1, 14 pages.

Abstract : Let n be a positive integer and α_n be the arithmetic function which assigns the multiplicative order of a^n modulo n to every integer a coprime to n and vanishes elsewhere. Similarly, let β_n assign the projective multiplicative order of a^n modulo n to every integer a coprime to n and vanishes elsewhere. In this paper, we present a study of these two arithmetic functions. In particular, we prove that for positive integers n_1 and n_2 with the same square-free part, there exists an exact relationship between the functions α_{n_1} and α_{n_2} and between the functions β_{n_1} and β_{n_2}. This allows us to reduce the determination of α_n and β_n to the case where n is square-free. These arithmetic functions recently appeared in the context of an old problem of Molluzzo, and more precisely in the study of which arithmetic progressions yield a balanced Steinhaus triangle in $\mathbb{Z}/n\mathbb{Z}$ for n odd.

MSC2000: 11A05, 11A07, 11A25.

Keywords: multiplicative order, projective multiplicative order, balanced Steinhaus triangles, Steinhaus triangles, Molluzzo's Problem.

3. Regular Steinhaus graphs of odd degree, Discrete Math. **309** (13), pp. 4545–4554, 2009.

Abstract : A Steinhaus matrix is a binary square matrix of size n which is symmetric, with a diagonal of zeros, and whose upper-triangular coefficients satisfy $a_{i,j} = a_{i-1,j-1} + a_{i-1,j}$ for all $2 \leqslant i < j \leqslant n$. Steinhaus matrices are determined by their first row. A Steinhaus graph is a simple graph whose adjacency matrix is a Steinhaus matrix. We give a short new proof of a theorem, due to Dymacek, which states that even Steinhaus graphs, i.e. those with all vertex degrees even, have doubly-symmetric Steinhaus matrices. In 1979 Dymacek conjectured that the complete graph on two vertices K_2 is the only regular Steinhaus graph of odd degree. Using Dymacek's theorem, we prove that if $(a_{i,j})_{1\leqslant i,j\leqslant n}$ is a Steinhaus matrix associated with a regular Steinhaus graph of odd degree then its sub-matrix $(a_{i,j})_{2\leqslant i,j\leqslant n-1}$ is a multi-symmetric matrix, that is a doubly-symmetric matrix where each row of its upper-triangular part is a symmetric sequence. We prove that the multi-symmetric Steinhaus matrices of size n whose Steinhaus graphs are regular modulo 4, i.e. where all vertex degrees are equal modulo 4, only depend on $\lceil n/24 \rceil$ parameters for all even numbers n, and on $\lceil n/30 \rceil$ parameters in the odd case. This result permits us to verify Dymacek's conjecture up to 1500 vertices in the odd case.

Keywords : Steinhaus graph, Steinhaus matrix, Steinhaus triangle, Regular graph, Regular Steinhaus graph, Dymacek's conjecture.

4. On a problem of Molluzzo concerning Steinhaus triangles in finite cyclic groups, Integers **8** (2008), Article A37, 29 pages.

Abstract : Let X be a finite sequence of length $m \geqslant 1$ in $\mathbb{Z}/n\mathbb{Z}$. The *derived sequence* $\partial X$of X is the sequence of length $m-1$ obtained by pairwise adding consecutive terms of X. The collection of iterated derived sequences of X, until length 1 is reached, determines a triangle, the *Steinhaus triangle* ΔX *generated by the sequence* X. We say that X is *balanced* if its Steinhaus triangle ΔX contains each element of $\mathbb{Z}/n\mathbb{Z}$ with the same multiplicity. An obvious necessary condition for m to be the length of a balanced sequence in $\mathbb{Z}/n\mathbb{Z}$ is that n divides the binomial coefficient $\binom{m+1}{2}$. It is an open problem to determine whether this condition on m is also sufficient. This problem was posed by Hugo Steinhaus in 1963 for $n = 2$ and generalized by John C. Molluzzo in 1976 for $n \geqslant 3$. So far, only the case $n = 2$ has been solved, by Heiko Harborth in 1972. In this paper, we answer positively Molluzzo's problem in the case $n = 3^k$ for all $k \geqslant 1$. Moreover, for every odd integer $n \geqslant 3$, we construct infinitely many balanced sequences in $\mathbb{Z}/n\mathbb{Z}$. This is achieved by analysing the Steinhaus triangles generated by arithmetic progressions. In contrast, for any n even with $n \geqslant 4$, it is not known whether there exist infinitely many balanced sequences in $\mathbb{Z}/n\mathbb{Z}$. As for arithmetic progressions, still for n even, we show that they are never balanced, except for exactly 8 cases occurring at $n = 2$ and $n = 6$.