Report on Research Accomplished

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My interests primarily lie in Design and Analysis of Algorithms and Game Theory, especially in Algorithmic Game Theory, Approximation and Online Algorithms. More specifically, I am interested in the existence and inefficiency of pure Nash equilibria in games and mechanisms, as well as in solutions to improve the quality of games' outcomes. An exciting chunk of my work is the study of algorithmic mechanism design with desired properties such as truthfulness, profit maximizing. My interests include designing approximation algorithm and online algorithm for combinatorial optimization problems in Scheduling, Networks, etc.

Existence of pure Nash equilibria and the inefficiency In a game, a *mixed* Nash equilibrium – where players choose a distribution over strategies and no one can get a better payoff by unilaterally changing his mixed strategy – always exists [6]. A *pure* Nash equilibrium in these games is a strategy profile in which each player deterministically chooses a strategy and the strategy profile is resilient to deviation. As contrast to the mixed equilibrium, the pure one does not always exist. A natural question raised when studying a game is whether the game possesses pure equilibria.

As our work shows [2, 7], deciding the existence of pure equilibrium in many games is $\mathcal{NP}$-hard. The technique is the reduction primarily based on *negated* gadget – which is the construction of an instance of the game with no pure equilibrium. In the positive angle, the existence of equilibrium is usually proved by using a potential argument with respect to the *best-response dynamic* – among players who can increase their payoff, let an arbitrary player take the best strategy. In our work [2, 3], we also follow this scheme in proving the existence of equilibrium. Nevertheless, a game with no potential function does not imply that there is no equilibrium in the game. In [3], we refine the best-response dynamic and construct a *novel* potential function with respect to this dynamic in order to prove the equilibrium existence. Informally, in a potential game, one can get an equilibrium by following an arbitrary orbit. In our proof, the dynamic follows a particular orbit and converges to an equilibrium. We believe that the dynamic together with the new potential function are useful in proving the existence of pure equilibrium in different games and of independent interests.

We study the inefficiency of equilibria. Specifically, we study two well-known measures: price of anarchy, denoted as PoA (the price of stability, denoted as PoS), which are the ratio between the worst (best) equilibrium and the optimum. Moreover, we introduce and study a new measure, called *social discrepancy* [2] which is the ratio between the worst and the best pure equilibrium. Note that the social cost discrepancy is not the ratio between the PoA and the PoS. The idea is that a small social cost discrepancy guarantees that the social costs of Nash equilibria do not differ too much, and measures a degree of choice in the game. Additionally, in some settings it may be unfair to compare the cost of a Nash equilibrium with the optimal cost, which may not be attained by selfish agents. It is worth to mention that from our introduction of Voronoi Games on graphs [2], the model and its variant are well considered and motivate, inspire new works [5, 4, 8].

Coordination mechanism With the development of the Internet, large-scale autonomous systems became more and more important. The systems consist of many independent and selfish agents who compete for the usage of shared resources. Every configuration has some social cost, as well as individual costs for every agent. Due to the lack of coordination, the equilibrium configurations may have high cost compared to the global social optimum. Since the behavior of the agents is influenced by the individual costs, it is natural to come up with a *coordination mechanisms* that both force the existence of Nash equilibria and reduce the price of anarchy. The idea is to reflect the social cost in the individual costs, so that selfish agents' behaviors result in a socially desired solution.

We studied coordination mechanisms for Scheduling Games in which each player is a job and players choose a machine which minimizes their costs, defined as the completion times of jobs. In the game, machines have policies that decide how to schedule jobs. There are two studied models: (i) *strongly local policies* means that a machine looks only at the processing time of jobs on it; (ii) *local policies* means that a machine knows all information concerning the jobs assigns to it. But, how machines schedule jobs if they learn nothing about jobs or if jobs report false information in order to get a lower cost. In [3], we propose a coordination mechanism based on a *non-clairvoyant* policy, called EQUI. Roughly speaking, EQUI schedules the jobs in parallel preemptively using time-multiplexing and assigns to every job the same fraction of the CPU. As the results, the game under EQUI policy always possesses pure Nash equilibrium and the PoA of the game is the same as the PoA induced by the best strongly local policy one can expect. The mechanism naturally turns out to be *truthful* – everyone has an incentive to report the true information.

Online mechanism design The goal of this domain is to design a mechanism, in which the underlying data is unknown, that interacts with rational participants so that self-interest behavior of participants yields a desirable outcome. Online mechanism design extends the domain to dynamic settings. Decisions in these settings must be made without knowledge of future information in the sense of online algorithms. Online auctions, such as the phenomenally-fast growing Adwords of Google, Yahoo!, ... or airline tickets markets are the most obvious and interesting motivations for this field.

Consider a production site that produces some perishable good and at a regular rate, for every time unit, one item is produced. These items have to be immediately delivered to some customer, as they cannot be stored, consider for example electricity or ice-cream. In this scenario, the customers arrive online and they are *single-minded.* Every customer i arrives at some release time r_i and announces (in a single-minded sense) that he would pay w_i Euros if he gets p_i items before the deadline d_i, otherwise he pays nothing. The goal is to (i) maximize the *welfare* – which is the total value w_i obtained from the satisfied customers and (ii) design a competitive mechanism such that customers have an incentive to report their true value.

As the results, in [1], we design an optimal truthful mechanism which is $\Theta(k/\log k)$-competitive where $k = \max_i p_i$. In case all customers have the same demand (i.e., $p_i = k\ \forall i$), there is a 5-competitive truthful mechanism.

References

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