Cost allocation protocols for network formation on connection situations

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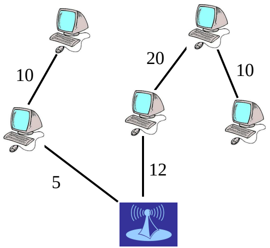
Summary

Building a network in a strategic setting

Situation: a set of agents building a network in order to be connected to a given source.



Assumptions:

- making a link $e = (i, j)$ has a cost/consumes energy $w(e)$;
- non cooperative game: agents do not make binding agreements on the design of the network. Each agent wants to minimize its own cost.

Question and objectives

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$\rightarrow$ address the problem of the design of cost allocation protocols to coordinate players placed on the nodes of a graph in such a way that:

- convergence under Better Response Dynamics (BRD) holds
- an efficient (minimum cost) communication network is built.

The strategic game

- $G = (N', E, w)$ is an undirected, connected and weighted graph, where $N' = N \cup \{0\}$ and $N = \{1, 2, \ldots, n\}$, and $w(e)$ is the cost/power needed to build/use the link e.

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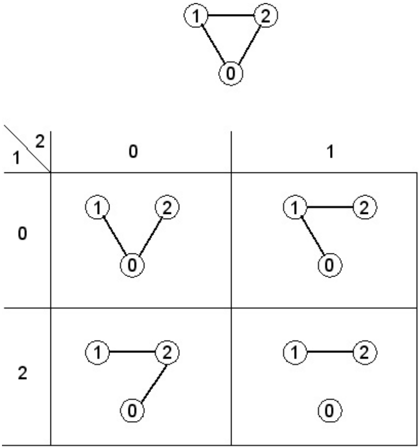
- The cost of remaining disconnected from the source is infinite.

An example of game form with two players



Some properties for protocols and games

- A cost allocation protocol c such that $\sum_{i \in S} c_i(w, S) = w(T_S)$ for every strategy profile S is said budget-balanced.
(for connected players if some players are not connected)
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A protocol is said state-dependent iff for every state S and every weight functions w, w', with $w(e) = w'(e)$ for every $e \in T_S$, then $c_i(w, S) = c_i(w', S)$ for every $i \in S$.

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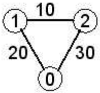

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0	(20, 30)	(20, 10)
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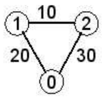

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Nash equilibrium $(0, 1)$ is optimal (i.e. $T_{(0,1)}$ is a minimum cost spanning tree (mcst) connecting all nodes in N'), but $(2, 0)$ is not.

Better Response Dynamic and Optimality

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- A Better Response Dynamic (BRD, also called Nash dynamics) (associated with a protocol c) is a sequence of states $S^0, S^1, \ldots,$ such that each state S^k (except S^0) is resulted by a better response of some player from the state S^{k-1}.

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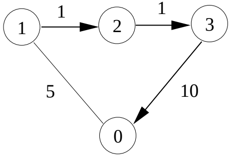
Theorem If a protocol is budget-balanced and optimal, then it is not state-dependent.

A budget-balanced and optimal protocol

Idea: If the network is not optimal (extra cost Δ), charge this cost Δ to a player (a set of players) to create for them an incentive to change.
Problem: find who should pay!



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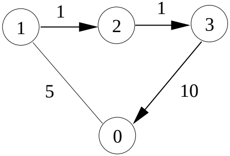

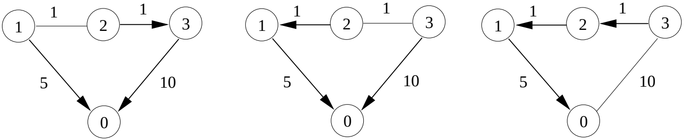

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Two protocols:

- One fairly (equally) shares cost between players in a optimal situation;
- One is more like Bird's protocol (players pay one link).

Ordinal potential game

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Thanks to a potential function $\Phi(S)$, ie Φ only decreases during a BRD.
$\Phi(S) = (|N \setminus \texttt{con}(S)|, |\hat{V}(S)|, \sum_{i \in \hat{V}(S)} |E_S(i)|)$
where $E_S(i) = \{j \in N' : w(i,j) < w(i, S_i)\}$.

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- the inherent limitations of the optimal protocols proposed in this paper is that it depends on the choice of an *a priori* selected mcst.